JUNIOR HIGH-SCHOOL MATHEMATICS[1]

E. R. BRESLICH
School of Education, University of Chicago

Within the last thirty years junior high schools have been started all over this country. They aim to break away from the traditions of the past and to reorganize the subject-matter from a social standpoint, based upon sound pedagogical and psychological principles.

Historically our education really began with an organization inspired by the educational institutions of Europe. This gave us three types of schools:

1. The common, or district school, a school for the common people.

2. The college and college preparatory schools. They are of English ancestry and essentially aristocratic.

3. The academy, an upper school inaugurated by the middle class to improve on the district school. It was often college preparatory and finally it either changed to a private preparatory school or a high school.

The system creates class distinction from which we have never really broken away. It is undemocratic.

The graded school was created by a broad democratic movement. It undertook to replace the common school, but because it was borrowed from Europe and was rigid in its organization it has remained in reality an unassimilated factor in American education.

The junior high school plans to eliminate the break between the eighth grade and the high school. Broadly speaking, it combines into one school the seventh, eighth, and ninth grades, or two years of elementary school and one of the high school, thus leaving six grades to the elementary school in which to acquire the elementary concepts and the tools of learning. It aims to break away from the lines laid down at the time the graded schools came into existence. It is an Americanizing movement and one of the most promising movements in education.

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There is much to be said in favor of a plan which makes the most important change come at the end of the sixth year rather than at the eighth. For the pupil who continues his studies in the high school as well as for those who do not continue, it is of most importance to be thoroughly grounded in the fundamentals. Six years are quite sufficient to give him mastery of this work. Indeed, evidence tends to show that if pupils were promoted directly from the sixth grade to the high school, they would be able to carry the work successfully if proper methods of instruction were employed. For example, for each year during the last five years one group of pupils coming from the sixth grade of the elementary school of the School of Education of the University of Chicago has taken up the same work in first-year mathematics as the pupils coming from the eighth grade of the elementary schools of the city. The quantity and quality of the work done by this group during the first year are quite as satisfactory as in the other first-year classes. The noticeable differences are, on one side, the greater enthusiasm shown by the seventh-grade group and their marked eagerness to learn, and, on the other hand, the tendency to be superficial, with the result that they forget more easily what they have learned. With proper methods of instruction, if the work is presented to them concretely, and their studying directed carefully, most of it being done in the classroom under the supervision of the teacher, they are able to do and to retain the work now offered in the first year of the high school. In the second-year course this group is distributed among the regular second-year classes. They have shown that they are fully as able as the other pupils to carry the second course and also those of the succeeding years. In this year's senior class they are found among the leaders of the class, both in scholarship and in social activities, and I have already received word from two of our last year's graduates, one in the University of Wisconsin and one in Boston Institute of Technology, to the effect that they are leading their classes in mathematics.

I do not wish to be understood as advocating that we eliminate the seventh and eighth grades and take up the work of the high school at the end of the sixth grade. If high-school preparation were the only justifiable end of the elementary school, this would be worth trying. The real problem is to plan a course which will meet the dominant interest and mental capacity of children during a certain period of growth which begins at about the twelfth year.

The pupil's physical nature is changing. Growth begins to be accelerated. There is an abundance of uncontrolled energy. Boys and girls at this age become highly nervous and a different treatment becomes necessary. They need much outdoor activity. They should therefore have an easy school schedule and little or no home work. Their whole course of study must conform largely to the demands of nature.

Another good reason for having the break between the elementary school and junior high school come at the end of the sixth grade is the interesting fact that twelve-year mentality, as shown by mental tests, is about the point where genuine education can begin—where the individual becomes adjustable to modern civilization. The pupil's mental development is now undergoing a change. The completion of the sixth year of school life marks the close of a distinct period of child life. Normal children at this age begin to develop generalizing power and to be able to do school work that demands it. They begin to form judgments and to inquire for reason and must have material upon which to exercise these powers. It is very important, therefore, that the change from the elementary school to the junior high school should come at the beginning rather than during this period.

Furthermore, this is a period of receptivity. Hence, in their school work these pupils should be given much content and a wide experience. They should thus lay the foundation on which to build their future education.

It is seen that the break between the eighth grade of the grammar school and the high school comes at an unfortunate time for the pupil. The high school starts at no definite point and terminates at no definite point. Adolescence which normally begins with the twelfth year should determine the period of secondary education.

Since a course of study in a subject must conform largely to the demands of nature we cannot proceed arbitrarily in planning the school work for this new institution. For example, in mathematics we cannot simply transfer the material now offered in the senior high school and give courses in abstract algebra and demonstrative geometry. Nor can we expect to be successful by using the methods of the high school. It is altogether possible that some of the work now offered in the high school should be brought down, that some of the material now taught in the seventh and eighth grades of the

elementary school should be retained, and that some of the typical work of these grades should even be transferred to the senior high school. What is really needed is a great deal of research for creating new material in addition to the arithmetic, algebra, and geometry now found in our courses in mathematics. In working out a program we must understand boys and girls of the age entering this period and provide the work best adapted to their needs, abilities, and natural inclinations. We must understand the purposes and limitations of the junior high school; we must know when it begins and how far it is to extend.

There are now a number of textbooks on junior high-school mathematics on the market. In general they comprise the rudiments of arithmetic, algebra, and geometry. A course in arithmetic is usually given first, followed by concrete geometry, after which the study of algebra is taken up. Let us see to what extent such courses satisfy the demands of this period.

It is assumed that the fundamentals of arithmetic needed in the future work have been sufficiently covered in the first six years. We may further assume that this work has been done sufficiently well. Under those conditions we may ask whether a review of arithmetic at the beginning of the course is necessary and worth while. Some years ago the department of mathematics in the University High School felt a need of such a review even for pupils who had completed the eighth grade. A two weeks' review course was planned, only such material being selected as the pupil would meet in his high-school work. This was done by simply omitting the letters in the algebraic problems found in the first-year textbook. In other words the pupils were drilled on the identical arithmetical work which they were to meet in the course in algebra. The results were most disappointing. It was found later that the pupils who had this review were no better able to avoid arithmetical mistakes than former classes. It became evident that thoroughness could not be secured by duplication or repetition. Moreover, drill on elements previously mastered is monotonous and the work is of questionable value. It is a waste of time at the expense of the new work which should be begun as soon as possible. If reviews are needed they must be given by offering opportunities to apply the fundamentals to some useful end, in connection with problems or topics of interest to the pupil. Indeed, some of these junior high-school books contain a wealth of

such problems. Whenever it becomes apparent in the solution of problems that pupils have forgotten certain processes and that they need further drill, that is the time and place to give it, not at the beginning of the course. By thus meeting these arithmetical computations in a large number of new situations, pupils will get a review with a new view. They will not be allowed to forget what they know and will learn to eliminate arithmetical errors. Teachers must therefore welcome frequent opportunities for arithmetical computations. These should always be performed by every pupil in the classroom at his seat and not simply be worked out by one pupil at the blackboard or explained by the teacher. In fact, practice should be kept up throughout the senior high school, and not only in classes in mathematics. Other subjects such as the sciences, where arithmetic is needed to solve problems, should share in the responsibility to give the pupils a wide experience in arithmetical computations.

A similar situation is found in the junior college. College teachers complain that students do not know the fundamentals of high-school mathematics, such as the solution of a quadratic equation. The reason for this is not that the solution of such equations was not taught well in the high school, but that it has not been met in a sufficiently large number of situations.

Even in the high school we find that textbooks for second courses in algebra usually begin with a review of the fundamentals of the first year's work. The same thing occurs in courses in college algebra. All along the line these review courses accomplish little and cause much waste of time.

A second striking feature of the present junior high-school texts is that advanced work in arithmetic precedes the study of geometry and algebra. The principal argument in favor of this arrangement seems to be that there is a decided advantage in having the pupil feel that the work he is about to study in the junior high school is after all not so very different from the preceding work of the sixth grade. However, elementary-school pupils look toward the junior high school as a school which is quite different from the elementary school. They expect, therefore, to take up the study of new subjects. Hence, many of them, especially those who found arithmetic difficult, are greatly disappointed upon finding that they are to have the same type of work in this new school. They would take up a different type of work with more interest and enthusiasm.

Therefore, let there be just enough incidental arithmetic in the first-year course to maintain the pupil's efficiency in calculation, and let the topical study of this subject be postponed to a later period. Another reason for this postponement is the fact that some phases of the course in arithmetic involve more difficult reasoning than the fundamentals of algebra and geometry and may better be given after pupils have become familiar with the elements of these subjects, and when their reasoning powers are better developed.

At present there is a relatively excessive amount of time given to arithmetic, which contains much that is unessential or fundamentally beyond the pupils' experience. Such topics as stocks and bonds, exchange, compound interest, partial payment are things that are foreign to the pupil at this age and are seldom touched upon in future courses. They may well be transferred to the upper classes of the high school, where they should be made elective for those who are interested in commercial work.

We have said that the junior high-school period is a time of experience-getting. Hence the arithmetic studied should relate to things of real interest to the pupil. It should be subsidiary to larger occupational interests. It should be arithmetic of the store, the farm, and industry. It should deal with matters of the home, such as planning family expense accounts on a certain income, saving to buy a lot on an installment plan, incomes from boys' activities. Most of the junior high-school textbooks contain an abundance of problems of the types which provide for the general needs of immediate, not remote, life utilization. Furthermore, the work in geometry and algebra given in these textbooks offers many opportunities and motives for the study of arithmetic. There is a marked agreement as to the correlation of geometry and arithmetic, and algebra and arithmetic. However, the correlation of geometry and algebra is comparatively small.

Assuming that our interpretation of the junior high-school period is correct, and that this is a period of experience-getting, this experience should extend over the whole field of elementary mathematics, arithmetic, algebra, geometry, and even trigonometry. These subjects are not to be taught separately, at least not during the first two years. What is wanted is a wide experience with the mathematical concepts which the pupil builds into a growing structure. All of these subjects contain simple facts which can

easily be mastered by the twelve-year-old pupil. Geometry is experience-getting in space relations; algebra, in abstract quantity relations. Experience gained in either should be used whenever it can be made helpful in the study of the other. When mathematics is taught essentially as one subject, it is possible to keep the important principles as far as possible under review, thus increasing the power of retention. Algebra is not to be taught as an organized science and the "sacred sequence" of Euclid in geometry will have to be abandoned. Indeed this sacred sequence does not longer exist even in the ordinary course in demonstrative geometry. For example, Euclid proved his constructions before he used them. Today every textbook introduces these constructions early and gives the proof later. The real value which the pupil derives from the study of geometry depends more upon the progress he makes than upon the requirements of a rigorous logical arrangement. Similarly, algebra taught as an organized science is to a pupil at this age nothing but a mechanical juggling of symbols, a wearisome iteration of meaningless manipulation, a waste of his time which stunts his intellectual growth. Much of the difficulty in the later courses is due to the fact that the fundamental concepts have never been clear and that too brief a period of time has been allowed the pupil to get clear-cut notions about definite things. As a result much of his knowledge is vague and indefinite. Giving the pupil a wide experience with these fundamentals will make possible a gradual and easy approach to the parts of the subject that are later taught in the high school.

Such material should be selected as proves its worth by actual service in the life of the pupil and such facts should be included as the pupil needs in his other studies. The material is to be arranged as far as possible in psychological order. New terms are to be learned through use and as need for them arises and the work progresses. The method of beginning with definitions and rules makes a course hard to comprehend, is repulsive to the young learner, and should therefore be avoided.

The geometry of the junior high school is not to be demonstrative but the geometry of the classroom, home, field, and park. Intuitional, mensurational, and constructive geometry furnishes the most concrete material we may offer to the pupil. Much of this should be given in the first year. The facts must appeal to the

pupils as valuable information. Free use is to be made of the ruler, compass, and protractor, to give the pupils a chance to be doing things. They measure, they estimate, they feel that the study is a real thing. They discover many new facts by examining the properties of their drawings. They work with them until they are thoroughly familiar, and thus form a basis on which they may build the theoretical work to follow. The introduction to demonstrative geometry should be very gradual. Pupils will reason about geometrical facts because they seem worth reasoning about. They will work hard because boys and girls like hard work, if they see the purpose of it and have a clear understanding of the ends to be attained. They know how to prove things long before they can demonstrate. The first proofs, although not of scientific rigor nor of the form demanded later, are thoroughly convincing. The form gradually grows as need arises. Thus it is possible to develop gradually an appreciation of the need for a demonstrative proof. At first they verify the theorem by measurement. When they understand the difficulty of making accurate measurements they learn to employ informal reasoning in some simple problems. When they realize the danger in incorrect reasoning and feel the needs for accuracy of statement, they are ready to learn to give a rigorous proof. A year's time may be given to the development of each of these abilities.

The algebra of the junior high school should be the outcome of concrete problems. The formulas should be related to real things. Formulas taken from the shop, the trade journals, engineering pocketbooks can be made interesting. Formulas may be used to show the advantage of the algebraic method over the arithmetical method in problems of percentage, discount, and interest. The equation which is the core of the course is a tool for problem-solving. There is to be no symbol juggling. The rules, principles, and processes are rationalized. The operations are a means for solving equations and are introduced as needed.

Geometry will furnish an abundance of concrete material. It should be used freely to make clear abstract algebraic principles which otherwise remain hazy. Many theorems, such as the theorem of Pythagoras, are essentially numerical relations. They furnish formulas for algebraic solution and lend themselves easily to algebraic treatment.

Graphical representation is to be used freely. To avoid an unnecessary accumulation of difficulties, positive and negative numbers are to be studied after the operations with positive literal numbers have been mastered. Much unnecessary material should be omitted.

The quantity of work done in the junior high school should be such that the pupil who enters the senior high school should have had a sufficient amount of work to enable him to complete during the first year the ordinary plane geometry, a considerable amount of solid geometry, and algebra through quadratics.

The methods of teaching and the subject studied gradually should change from those of the elementary school into those of the high school. At first all of the work, later most of it, is to be done in the classroom under the direction of the teacher. The pupil is to be shown how to study in order that he may accomplish his daily task in the time allowed for it in the classroom. The project method is to be used freely because it supplies a strong motive. For this reason, by far the greatest part of the work should be the solution of algebraic, geometric, and arithmetical problems.

For the three years of the junior high school the purely mathematical material of the program should be about as follows.

First Year

Geometry:

Simple graphs used to interpret data, to illustrate algebraic processes, to motivate the study of fractions and the metric system.
Angles and the use of the protractor in drawings.
Accurate drawings according to given directions.
Use of compasses in fundamental constructions.
Parallel, intersecting, perpendicular lines.
Plane figures are studied because they are found in the classroom and elsewhere, e.g., on the familiar solids.
Areas, perimeters of triangles, rectangles, parallelograms, trapezoids, and circles.
Volumes of parallelopiped, cylinder.
Some simple reasoning processes.

Algebra:

Symbols of algebra.
Simple equation in one unknown, used in stating and solving problems.
Formulas of mensuration, shop, trade journals.

Arithmetic:

The geometry furnishes problems in common fractions and in decimal fractions, e.g., scale drawings create need for operations with fractions.

Study of rectangles leads to such problems as plastering, painting, division of land, ladders, rafters, baseball diamonds, indoor surveying, etc. Study of the circle leads to such problems as flower bed, wheels, cistern, race track, parcel post zones. The formula leads to problems of indirect measurement, percentage, interest, discount.

SECOND YEAR

Geometry:

Graphs to illustrate positive and negative numbers, statistics taken from newspapers, almanacs.

Similar figures, problems in heights and distances. Scale drawings. Table of tangents.

Construction of designs.

Many principles of angles and polygons to furnish problems for algebraic solution.

These principles are discovered by accurate construction of figures and by measurement.

Algebra:

The operations as needed in the solution of equations and manipulation of formulas.

Positive and negative numbers.

The equation as a tool for problem-solving.

Fractions of a simple type.

Ratio.

Proportion.

Arithmetic:

Essentials to be reviewed in connection with algebraic topics. Algebraic processes are begun with a review of related arithmetic processes.

Evaluation of formulas.

Fractions and decimals as coefficients in equations.

Pre-estimating results before calculating.

Business aspects of arithmetical applications relating to actual experience.

Problems of percentage, interest, insurance, discount, account.

Problems of the home, school, community, industries.

THIRD YEAR

Geometry:

Congruent triangles. Informal reasoning as a period of transition to formal proofs. Symmetry. Considerable geometry of space.

Theorems and problems of the isosceles and equilateral triangle. Some theorems on circles.

Constructions. Designs.

Proofs of the fundamental constructions.

Accurate drawings of figures from given parts.

Algebra:

Simultaneous linear equations solved by graph and by one of the methods of elimination.

Some factoring: common factor; difference of two squares; trinomial: ax^2+bx+c.

Difficult parts of algebra, such as complicated factoring problems, complex fractions, etc., are omitted.

Variation; formulas.

Quadratic equations solved graphically and by factoring.

Arithmetic:

Final treatment of the more difficult subjects.

Square root of arithmetical numbers.